SAFE HANDS & IIT-ian's PACE

LEAP TEST# 12 (JEE) ANS KEY Dt. 01-01-2024

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	A	31	B	61	C
2	B	32	C	62	A
3	D	33	A	63	C
4	D	34	A	64	A
5	B	35	A	65	A
6	B	36	D	66	A
7	D	37	A	67	B
8	D	38	A	68	A
9	A	39	B	69	C
10	A	40	D	70	A
11	D	41	B	71	A
12	C	42	B	72	C
13	A	43	B	73	B
14	C	44	D	74	A
15	A	45	B	75	B
16	B	46	B	76	A
17	A	47	C	77	B
18	D	48	D	78	A
19	D	49	A	79	C
20	B	50	B	80	C
21	0	51	0	81	0
22	2	52	5	82	4
23	3	53	2	83	7
24	3	54	2	84	2
25	1	55	2	85	1

See Maths solutions on next page.....

: ANSWER KEY LT 12 :

61)	c	62)	a	63)	c	64)	a	77)	b	78)	a	79)	c	80)	c
65)	a	66)	a	67)	b	68)	a	81)	0	82)	4	83)	7	84)	2
69)	c	70)	a	71)	a	72)	c	85)	1						
73)	b	74)	a	75)	b	76)	a								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(c)**

Given differential equation is

$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ ...(i)

(a) $y = 2 \Rightarrow \frac{dy}{dx} = 0$

On putting in Eq. (i),

$0^2 - x(0) + y = 0$

$\Rightarrow y = 0$ which is not satisfied.

(b) $y = 2x \Rightarrow \frac{dy}{dx} = 2$

On putting in Eq. (i),

$(2)^2 - x.2 + y = 0$

$\Rightarrow 4 - 2x + y = 0$

$\Rightarrow y = 2x$ which is not satisfied.

(c) $y = 2x - 4 \Rightarrow \frac{dy}{dx} = 2$

On putting in Eq.(i)

$(2)^2 - x - 2 + y$

$4 - 2x + 2x - 4 = 0$ $[\because y = 2x - 4]$

$y = 2x - 4$ is satisfied

(d) $y = 2x^2 - 4$

$\frac{dy}{dx} = 4x$

On putting in Eq. (i),

$(4x)^2 - x.4x + y = 0$

$\Rightarrow y = 0$ which is not satisfied

62 **(a)**

$y'y''' = 3(y'')^2$

$\Rightarrow \int \frac{y'''}{y''}dx = 3\int \frac{y''}{y_1'}dx$

$\Rightarrow \ln y'' = 3\ln y' + \ln c$

$\Rightarrow y'' = c(y')^3$

$\Rightarrow \int \frac{y''}{(y')^2}dx = \int cy'dx$

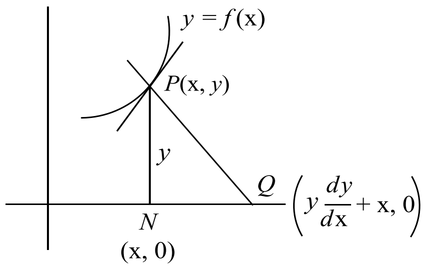
$\Rightarrow -\frac{1}{y'} = cy + d$

$\Rightarrow -dx = (cy + d)dy$

$\Rightarrow -x = \frac{cy^2}{2} + dy + e$

63 **(c)**

Equation of normal at point $P(x, y), Y - y = -\frac{dy}{dx}(X - x)$



$NQ = y\frac{dy}{dx} = \frac{x(1 + y^2)}{1 + x^2}$

$\Rightarrow \frac{xdx}{1 + x^2} = \frac{ydy}{1 + y^2}$

$\Rightarrow \ln(1 + x^2) = \ln(1 + y^2) + \ln c$

$\Rightarrow 1 + y^2 = \frac{1 + x^2}{c}$

It passes through $(3,1) \Rightarrow 1 + 1 = \frac{1+(3)^2}{6} \Rightarrow c = 5$

$\Rightarrow$ curve is $5 + 5y^2 = 1 + x^2$ or $x^2 - 5y^2 = 4$

64 **(a)**

If $(0, k)$ be the centre on y-axis then its radius will be k as it passes through origin. Hence its equation is

$x^2 + (y - k)^2 = k^2$

Or $x^2 + y^2 = 2ky$ (1)

$\therefore 2x + 2y\frac{dy}{dx} = 2k\frac{dy}{dx}$

$= \frac{x^2 + y^2}{y}\frac{dy}{dx}$ [by (1)]

$\therefore 2xy = (x^2 + y^2 - 2y^2)\frac{dy}{dx}$

Or $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

65 **(a)**

$x\frac{dy}{dx} + y(\log y) = 0$

$$\Rightarrow \int \frac{dx}{x} + \int \frac{dy}{y(\log y)} = c$$
$\Rightarrow \log x + \log(\log y) = \log c$
$\Rightarrow x \log y = c$
$y(1) = e \Rightarrow c = 1$
Hence, the equation of the curve is $x \log y = 1$

66 **(a)**

$$\frac{dy}{dx} = \frac{1}{xy[x^2 \sin y^2 + 1]}$$
$$\Rightarrow \frac{dx}{dy} = xy\,[x^2 \sin y^2 + 1]$$
$$\Rightarrow \frac{1}{x^3}\frac{dx}{dy} - \frac{1}{x^2} y = y \sin y^2$$
Putting $-1/x^2 = u$, the least equation can be written as $\frac{du}{dy} + 2uy = 2y \sin y^2$
I.F. $= e^{y^2}$
$\therefore$ solution is $ue^{y^2} = \int 2y \sin y^2 e^{y^2} dy + C$
$$= \int (\sin t) e^t\, dt + C$$
$$= \frac{1}{2} e^{y^2} (\sin y^2 - \cos y^2) + c'$$
$$\Rightarrow 2u = (\sin y^2 - \cos y^2) + 2Ce^{-y^2}$$
$$\Rightarrow 2 = x^2[\cos y^2 - \sin y^2 - 2Ce^{-y^2}]$$

67 **(b)**

$$f''(x) - \frac{2x(x+1)}{x+1} f(x) = \frac{e^{x^2}}{(x+1)^2}$$
$\text{I.F.} = e^{\int -2x dx} = e^{-x^2}$
$\therefore$ solution is $f(x)e^{-x^2} = \int \frac{dx}{(x+1)^2} + C$
$$\Rightarrow f(x)e^{-x^2} = -\frac{1}{x+1} + C$$
Given $f(0) = 5 \Rightarrow C = 6$
$$\therefore f(x) = \left(\frac{6x+5}{x+1}\right) e^{x^2}$$

68 **(a)**

$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ (1)
Put $y = vx \;\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$
$\therefore$ equation (1) transforms to
$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2xvx} = \frac{1+v^2}{2v}$$
$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1-v^2}{2v}$$
$$\Rightarrow \frac{2vdv}{1-v^2} = \frac{dx}{x}$$
$\Rightarrow \log x + \log(1 - v^2) = \log C$
$\Rightarrow x(1 - v^2) = C$
$$\Rightarrow x\left(1 - \frac{y^2}{x^2}\right) = C$$
$\Rightarrow x^2 - y^2 = Cx$
It passes through (2, 1)
$$\therefore 4 - 1 = 2C \Rightarrow C = \frac{3}{2}$$
$$\therefore x^2 - y^2 = \frac{3}{2}x \Rightarrow 2(x^2 - y^2) = 3x$$

69 **(c)**

$$\frac{y'''}{y''} = 8 \Rightarrow \log y'' = 8x + c$$
When $x = 0, y'' = 1$ and $\log 1 = 0 \therefore c = 0$
$\therefore\; y'' = e^{8x}$. Integrating again
$y' = \frac{e^{8x}}{8} + \lambda$ when $x = 0, y'(0) = 0$
$\therefore \lambda = -1/8$
$\therefore\; y' = \frac{e^{8x}}{8} - \frac{1}{8}$. Integrate again
$$y = \frac{e^{8x}}{64} - \frac{x}{8} + k$$
Also when $x = 0, y = \frac{1}{8} \therefore k = \frac{7}{64}$
$$\therefore y = \frac{1}{8}\left(\frac{e^{8x}}{8} - x + \frac{7}{8}\right)$$

70 **(a)**

$(2y + xy^3)dx + (x + x^2y^2)dy = 0$
$\Rightarrow (2y\,dx + xdy) + (xy^3dx + x^2y^2dy) = 0$
Multiplying by x, we get
$(2xy\,dx + x^2\,dy) + (x^2y^3\,dx + x^3y^2dy) = 0$
$$\Rightarrow d(x^2y) + \frac{1}{3}d(x^3y^3) = 0$$
Integrating, we get $x^2y + \frac{x^3y^3}{3} = c$

71 **(a)**

Given, $\frac{dy}{dt} - \left(\frac{1}{1+t}\right) y = \frac{1}{(1+t)}$ and $y(0) = -1$
$\therefore \quad IF = e^{\int -\left(\frac{t}{1+t}\right)dt} = e^{-\int\left(1 - \frac{1}{1+t}\right)dt}$
$e^{-t + \log(1+t)} = e^{-t}(1+t)$
$\therefore$ Required solution is,
$ye^{-t}(1+t) = \int \frac{1}{1+t}\, e^{-t}\,(1+t)dt + c$
$= \int e^{-t}\; dt + c$
$\Rightarrow ye^{-1}(1+t) = -e^{-1} + c$
Since, $y(0) = -1$
$\Rightarrow \quad c = 0$
$\therefore \quad y = -\frac{1}{(1+t)}$
$\Rightarrow \quad y(1) = -\frac{1}{2}$

72 **(c)**

Let population $= x$, at time t years. Given $\frac{dx}{dt} \propto x$

$\Rightarrow \frac{dx}{dt} = kx$ where k is a constant of proportionality

Or $\frac{dx}{x} = kdt$. Integrating, we get $\ln x = kt + \ln c$

$\Rightarrow \frac{x}{c} = e^{kt}$ or $x = ce^{kt}$

If initially, i.e., when time $t = 0, x = x_0$

then $x_0 = ce^0 = c$

$\Rightarrow x = x_0 e^{kt}$

Given $x = 2x_0$ when $t = 30$ then $2x_0 = x_0 e^{30k} \Rightarrow 2 = e^{30k}$

$\therefore \ln 2 = 30k$ (1)

To find t, when t triples, $x = 3x_0$ $\therefore 3x_0 = x_0 e^{kt} \Rightarrow 3 = e^{kt}$

$\therefore \ln 3 = kt$ (2)

Dividing equation (2) by (1) then $\frac{t}{30} = \frac{\ln 3}{\ln 2}$ or

$t = 30 \times \frac{\ln 3}{\ln 2} = 30 \times 1.5849 = 48$ years (approx)

73 **(b)**

$(x^2 + xy)dy = (x^2 + y^2)dx \Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$

Let $\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ equation reduces to

$x\frac{dv}{dx} = \frac{1 + v^2}{1 + v} - v$

$= \frac{1 + v^2 - v - v^2}{1 + v}$

$= \frac{1 - v}{1 + v}$

$\Rightarrow \int \frac{1 + v}{1 - v} dv = \int \frac{dx}{x}$

$\Rightarrow -\int \left(1 - \frac{2}{1 - v}\right) dv = \int \frac{dx}{x}$

$\Rightarrow -v - 2\log(1 - v) = \log x + \log c$

$\Rightarrow -\frac{y}{x} - 2\log\left(\frac{x - y}{x}\right) = \log x + \log c$

$\Rightarrow \frac{-y}{x} - 2\log(x - y) = 2\log x = \log x + \log c$

$\Rightarrow \log x = 2\log(x - y) + \frac{y}{x} + k$ where $k = \log c$

74 **(a)**

$\frac{dV}{dt} = -k4\pi r^2$ (1)

But $V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ (2)

Hence, $\frac{dr}{dt} = -K$

75 **(b)**

$(x\cot y + \log\cos x)dy + (\log\sin y - y\tan x)dx = 0$

$\Rightarrow (x\cot y\, dy + \log\sin y\, dx) + (\log\cos x\, dy - y\tan x\, dx) = 0$

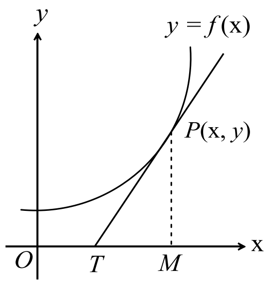
$\Rightarrow \int d(x\log\sin y) + \int d(y\log\cos x) = 0$

$\Rightarrow x\log\sin y + y\log\cos x = \log c$

$\Rightarrow (\sin y)^x(\cos x)^y = c$

76 **(a)**

Let the equation of the curve be $y = f(x)$



It is given that $OT \propto y$

$\Rightarrow OT = by$

$\Rightarrow OM - TM = by$

$\Rightarrow x - \frac{y}{dy/dx} = by$ [$\because TM =$ Length of the subtangent]

$\Rightarrow x - y\frac{dx}{dy} = by$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = -b$

It is linear differential equation

Its solution is $\frac{x}{y} = -b\log y + a$

$\Rightarrow x = y(a - b\log y)$

77 **(b)**

Putting $u = x - y$, we get $du/dx = 1 - dy/dx$. The given equation can be written as $1 - du/dx = \cos u$

$\Rightarrow (1 - \cos u) = du/dx$

$\Rightarrow \int \frac{du}{1 - \cos u} = \int dx + C$

$\Rightarrow \frac{1}{2}\int \operatorname{cosec}^2(u/2)du = \int dx + C$

$\Rightarrow x + \cot(u/2) = c$

$\Rightarrow x + \cot\frac{x - y}{2} = C$

78 **(a)**

$y = e^x(A\cos x + B\sin x)$

$\frac{dy}{dx} = e^x[-A\sin x + B\cos x] + e^x[A\cos x + B\sin x]$

$\frac{dy}{dx} = e^x[-A\sin x + B\cos x] + y \quad (1)$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = e^x[-A\sin x + B\cos x] + e^x[-A\cos x - B\sin x] + \frac{dy}{dx}$$

$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx} - y\right) - y + \frac{dy}{dx}$ [using (1)]

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

79 **(c)**

Given, $y = (c_1 + c_2)\cos(x + c_3) - c_4e^{x+c_5}$ ...(i)

$\Rightarrow \quad y = (c_1 + c_2)\cos(x + c_3) - c_4e^x.e^{c_5}$

Now, let $c_1 + c_2 = A, c_3 = B, c_4e^{c_5} = c$

$\Rightarrow \quad y = A\cos(x + B) - ce^x$ ...(ii)

On differentiating w.r.t.x, we get

$\frac{dy}{dx} = -A\sin(A + b) - ce^x$ ...(iii)

Again, on differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = -A\cos(x + B) - ce^x$ ...(iv)

$\Rightarrow \quad \frac{d^2y}{dx^2} = -y - 2\,ce^x$ ...(v)

$\Rightarrow \quad \frac{d^2y}{dx^2} + y = -2ce^x$

Again, on differentiating w.r.t. x, we get

$\frac{d^3y}{dx^3} + \frac{dy}{dx} = -2ce^x$ ...(vi)

$\Rightarrow \quad \frac{d^3y}{dx^2} + \frac{dy}{dx} = \frac{d^2y}{dx^2} + y$ [from Eq. (v)]

Which is a differential equation of order 3.

80 **(c)**

Slope of tangent $= \frac{dy}{dx}$

$\therefore$ slope of normal $= -\frac{dx}{dy}$

$\therefore$ the equation of normal is;

$$Y - y = -\frac{dx}{dy}(X - x)$$

This meets x-axis $(y = 0)$, where

$$-y = -\frac{dx}{dy}(X - x) \Rightarrow X = x + y\frac{dy}{dx}$$

$\therefore G$ is $\left(x + y\frac{dy}{dx}, 0\right)$

$\therefore OG = 2x \Rightarrow x + y\frac{dy}{dx} = 2x$

$\Rightarrow y\frac{dy}{dx} = x \Rightarrow y\,dy = xdx$

Integrating, we get $\frac{y^2}{2} = \frac{x^2}{2} + \frac{C}{2}$

$\Rightarrow y^2 - x^2 = c$, which is a hyperbola

Integer Answer Type

81 **(0)**

$y'(x) + y(x)f'(x) = f(x)f'(x)$ ... [Linear]

$\mathrm{I.F.} = e^{\int f'(x)dx} = e^{f(x)}$

General solution is

$$y(x)e^{f(x)} = \int e^{f(x)}f(x)f'(x)dx + c$$

$\Longrightarrow ye^{f(x)} = e^{f(x)}[f(x) - 1] + c$ ... (i)

Substituting x = 0, we get

$y(0)e^{f(0)} = e^{f(0)}[f(0) - 1] + c$

$\Longrightarrow c = e^0 = 1$

(i) reduces to

$ye^{f(x)} = e^{f(x)}[f(x) - 1] + 1$

Substituting x = 3, we get

$y(3)e^{f(3)} = e^{f(3)}[f(3) - 1] + 1$

$\Longrightarrow y(3) = e^0[0 - 1] + 1 = 0$

82 **(4)**

$$\frac{dy}{dx} - y = 1 - e^{-x}$$

$P = -1 \; Q = 1 - e^{-x}$

$\mathrm{I.F.} = e^{\int Pdx} = e^{\int -1dx} = e^{-x}$

$$\therefore y \cdot e^{-x} = \int e^{-x}(1 - e^{-x})dx + C$$

$$ye^{-x} = -e^{-x} + \frac{1}{2}e^{-2x} + C$$

$$y = -1 + \frac{1}{2}e^{-x} + Ce^x$$

$\because x = 0 \; y = y_0$

So $C = y_0 + \frac{1}{2}$

$$y = -1 + \frac{1}{2}e^{-x} + (y_0 + 1/2)e^x$$

$x \to \infty$ $y \to$ finite value so $y_0 + 1/2 = 0$

$y_0 = -1/2$

83 **(7)**

$$\frac{d^2y}{dx^2} = 6x - 4$$

Integrating with respect to x, we get

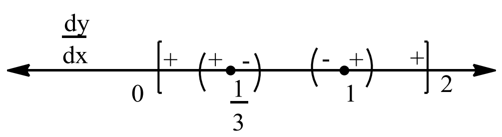
$$\frac{dy}{dx} = 3x^2 - 4x + c$$

$$\text{At } x = 1, \frac{dy}{dx} = 0 \implies c = 1$$

$$\implies \frac{dy}{dx} = 3x^2 - 4x + 1 \quad \ldots (i)$$

$$= (3x - 1)(x - 1)$$

$$\frac{dy}{dx} = 0 \implies x = \frac{1}{3}, 1$$



$x = \frac{1}{3}$ is a point of relative maxima

In $\left(0, \frac{1}{3}\right)$, the function is increasing.

In $\left(\frac{1}{3}, 1\right)$, the function is decreasing.

In $(1,2)$, the function is decreasing.

$\implies x = 0$ and $x = 2$ are points of relative minima

And relative maxima respectively.

Integrating (i), we get

$$y = x^3 - 2x^2 + x + k \quad \ldots (ii)$$

At $x = 1, y = 5$

$\implies k = 5$

(ii) reduces to

$$y = x^3 - 2x^2 + x + 5$$

At $x = 2, \quad y = 8 - 8 + 2 + 5 = 7$

$$\text{At } x = \frac{1}{3}, \ y = \frac{1}{27} - \frac{2}{9} + \frac{1}{3} + 5$$

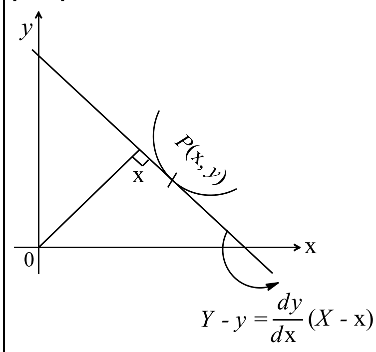
$$= \frac{1 - 6 + 9 + 135}{27} = \frac{139}{27}$$

$\implies$ Global maximum value $= 7$

84 **(2)**

Equation of tangent is $X\frac{dy}{dx} - y - Y\frac{dy}{dx} + y = 0$

perpendicular distance from origin is



$\therefore \perp$ from (0,0)$= x$

$$\left|\frac{0 - 0 - x\frac{dy}{dx} + y}{\sqrt{\left(\frac{dy}{dx}\right)^2 + 1}}\right| = x$$

$$\therefore \left|\frac{x\frac{dy}{dx} - y}{\sqrt{\left(\frac{dy}{dx}\right)^2 + 1}}\right| = x \Rightarrow \left(x\frac{dy}{dx} - y\right)^2$$

$$= x^2\left(1 + \left(\frac{dy}{dx}\right)^2\right)$$

$$\Rightarrow x^2\left(\frac{dy}{dx}\right)^2 + y^2 - 2xy\frac{dy}{dx} = x^2 + x^2\left(\frac{dy}{dx}\right)^2$$

$\Rightarrow \frac{y^2 - x^2}{2xy} = \frac{dy}{dx}$ (1) (Homogeneous)

Put $y = vx$ in (1)

$$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$\int \frac{2v}{v^2 + 1} dv = -\int \frac{dx}{x}$$

$$\ell n(v^2 + 1) = -\ell nx + \ell nc$$

$$v^2 + 1 = \frac{c}{x}$$

$$\frac{y^2 + x^2}{x^2} = \frac{c}{x} \Rightarrow y^2 + x^2 = cx$$

Passes through (1,1), then $c = 2$

$x^2 + y^2 - 2x = 0$

For intercept of curve on x-axis, put $y = 0$

We have $x^2 - 2x = 0$ or $x = 0,2$

Hence length of intercept is 2

85 **(1)**

$$\frac{dy}{dx} = \frac{1}{dx/dy}; \frac{d^2y}{dx^2} = \frac{d}{dy}\left(\frac{1}{dx/dy}\right) \cdot \frac{dy}{dx}$$

$$= -\frac{1}{(dx/dy)^3}\frac{d^2x}{dy^2}$$

Hence $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = 0$

Becomes $-x \cdot \frac{1}{(dx/dy)^3}\frac{d^2x}{dy^2} + \frac{1}{(dx/dy)^3} - \frac{1}{dx/dy} = 0$

Or $x\frac{d^2x}{dy^2} - 1 + \left(\frac{dx}{dy}\right)^2 = 0 \Rightarrow x\frac{d^2x}{dy} + \left(\frac{dx}{dy}\right)^2 = 1;$

$\therefore k = 1$